

Effect of magnetic fields on the spin evolution of non-polarized ^{87}Rb Bose-Einstein condensates

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The spin mixing dynamics of spin-1 Bose-Einstein condensates with zero magnetization and under an external magnetic field is investigated. The time-dependent solutions are obtained via a diagonalization of the Hamiltonian, which has a simple form under the single mode approximation. The features of evolution are compared in detail with those with the field removed so as to emphasize the effect of the field, which can induce strong oscillation in population of atoms in spin component 0. A new mode of oscillation characterized by a high frequency and a low frequency, is found when the field is sufficiently strong.

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I. INTRODUCTION

The rich physics in the Bose-Einstein condensate of atoms with spin degrees of freedom has attracted considerable interests in the last decade. [1, 2, 3, 4, 5, 6, 7, 25] Experimentally, one has been able to prepare spinor condensates of given number of atoms and given magnetization, and then to observe their evolutions. The time-dependent population of each spin component has been measured by the Stern-Gerlach splitting technique. Various modes of oscillation of the populations have been found, they are sensitive to the initial states. [8, 9, 10] Coherent dynamics has been observed experimentally. [11, 12, 13, 14]

When an external magnetic field is applied, the evolution will be affected. This is a way to control the evolution. Efforts along this line have been made. [15, 16, 17, 18, 19, 26] Most theoretical calculations on this problem are based on the mean-field theory. In this approach, the spin dynamics can be modeled as a non-rigid pendulum which displays a variety of periodic oscillations in magnetic fields. [20, 21] The dependence of the oscillation period on the magnetic field has been confirmed by experiments. [22] In fact, theoretical studies suggested that more complicate spin dynamic behavior would be possible. [10, 23]

In a previous paper of us [24], the spin evolution without a magnetic field has been studied. Where, instead of using the mean field theory, the time-dependent solutions of the Hamiltonian have been obtained under the single mode approximation (SMA). Thereby an analytical formula governing the evolution has been derived. The present paper is a direct generalization of the previous one, the aim is to clarify the effect of magnetic field on the spin evolution. For this purpose, the Hamiltonian arising from the SMA has been exactly diagonal-

ized. Thereby the time-dependent populations of spin-0 component starting from various initial states can be obtained. We have focused on the condensates with zero magnetization. Since the solutions of our Hamiltonian are exact, both short-term and long-term behaviors can be predicted by the theory. In the follows the basic approach is given in Section II. The features of spin-evolution without a magnetic field is briefly reviewed in Section III. Those under a magnetic field are presented in Section IV. In Section V, the effects of atom number and the confinement potential are discussed. Finally in Section VI, a summary is given.

II. HAMILTONIAN AND ITS SOLUTION

Let $U_{ij} = (c_0 + c_2 \mathbf{F}_i \cdot \mathbf{F}_j) \delta(\mathbf{r}_i - \mathbf{r}_j)$ be the spin-dependent interaction among N spin-1 atoms. Under the SMA, all the atoms have the same spatial normalized wave function $\phi(\mathbf{r})$. Accordingly, the Hamiltonian for the spin evolution under a magnetic field B with direction lying along the Z-axis reads

$$H = g\hat{S}^2 + q \sum_i F_{zi}^2 \quad (1)$$

where $g = \frac{1}{2}c_2 \int d\mathbf{r} |\phi(\mathbf{r})|^4$. The second term arises from the quadratic Zeeman effect, where $q = (E_+ + E_- - 2E_0)/2$ is the energy difference of the three Zeeman levels, which is related to B as $q = \mu_B^2 B^2 / (\hbar^2 E_{HFS})$, and E_{HFS} is the hyperfine splitting. The linear Zeeman term does not affect the dynamics due to the conservation of the total magnetization M .

Let a Fock-state be denoted as $|N_1, N_0, N_{-1}\rangle$, where N_μ is the number of atoms in spin component μ . Since $N_{\pm 1} = (N - N_0 \pm M)/2$, due to the conservation of N and M which are initially given, the Fock-state can be simply denoted as $|N_0\rangle$ with N_0 ranged from $N - M$, $N - M - 2$, to 0 or 1. They form a complete set of bases for the many particle spin space. The matrix elements of

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H read

$$\begin{aligned} \langle N'_0 | H | N_0 \rangle &= [gA_0(M, N_0) + q(N - N_0)]\delta_{N_0, N'_0} \\ &+ gA_+(M, N_0)\delta_{N_0, N'_0+2} \\ &+ gA_-(M, N_0)\delta_{N_0, N'_0-2} \end{aligned} \quad (2)$$

where

$$A_0(M, N_0) = M^2 + N + N_0 + 2NN_0 - 2N_0^2$$

$$A_+(M, N_0) = [N_0(N_0-1)(N+M-N_0+2)(N-M-N_0+2)]^{1/2} \pi / (|g|\omega)$$

$$A_-(M, N_0) = [(N_0+1)(N_0+2)(N+M-N_0)(N-M-N_0)]^{1/2}$$

After the diagonalization of the Hamiltonian, the eigenenergies E_j and eigenstates $|\phi_j\rangle$ can be obtained, and we have $H|\phi_j\rangle = E_j|\phi_j\rangle$, $|\phi_j\rangle = \sum_{N_0} c_{N_0}^j |N_0\rangle$. It is emphasized that both E_j and ϕ_j are exact for the Hamiltonian because the space for the diagonalization is complete.

Let the initial state be a Fock-state with given N , M , and $N_0|_{t=0} = I$, and is denoted as $|I\rangle$. Then the total spin-state at time t is

$$\begin{aligned} \Psi(t) &= e^{-iHt/\hbar} |I\rangle = \sum_j e^{-iE_j t/\hbar} |\phi_j\rangle \langle \phi_j | I \rangle \\ &= \sum_j c_I^j e^{-iE_j t/\hbar} |\phi_j\rangle \end{aligned} \quad (3)$$

It is emphasized that (3) is an exact time-dependent solution of the Hamiltonian, every detail of the evolution can be thereby extracted. We are interested in the t -dependence of the population of spin-0 component, namely, $\langle \Psi(t) | \hat{N}_0 | \Psi(t) \rangle / N \equiv \mathbf{P}_I^M(t)$, which can be derived from (3) as

$$\mathbf{P}_I^M(t) = \mathbf{B}_I^M + \mathbf{O}_I^M(t) \quad (4)$$

where

$$\mathbf{B}_I^M = \sum_j (c_I^j)^2 \sum_{N_0} (c_{N_0}^j)^2 N_0 / N \quad (5)$$

$$\mathbf{O}_I^M(t) = 2 \sum_{j < j'} \cos[(E_{j'} - E_j)t/\hbar] c_I^j c_I^{j'} \sum_{N_0} c_{N_0}^j c_{N_0}^{j'} N_0 / N \quad (6)$$

Eq.(4) implies an oscillation around the background \mathbf{B}_I^M .

We consider the condensate of ^{87}Rb atoms trapped by a harmonic potential $\frac{1}{2}m\omega^2 r^2$. In the present paper, $\hbar\omega$, mG and sec are used as units. For ^{87}Rb , $g \simeq -6.57 \times 10^{-5}(\omega/N^3)^{1/5}$ (evaluated under the Thomas-Fermi approximation), and $q \simeq 144\pi \times 10^{-6}B^2/\omega$. Numerical results with discussions are given in the follows.

III. EVOLUTION WITHOUT MAGNETIC FIELD

In order to understand the effect of magnetic field B , we first review the evolution with $B = 0$. The evolution is described by the formula of [24], which is exact for the Hamiltonian (1) with $q = 0$. In order to obtain numerical results from the formula, $N = 400$ and $\omega = 3000/\text{sec}$ are firstly assumed. Then the effect of N and ω will be studied. The evolution has the following features:

(i) \mathbf{P}_I^M is strictly periodic with a period $t_p = 2\pi/(|g|\omega)$ and symmetric with respect to $t_p/2$. $\mathbf{O}_I^M(t)$ is anti-symmetric with respect to $t_p/4$. Therefore, the study is sufficient to be confined in the duration $(0, t_p/4)$. Under the Thomas-Fermi approximation, $t_p \approx 1.521\pi(N/\omega^2)^{3/5} \times 10^4 \text{sec}$ (for ^{87}Rb). For the given N and ω , the above duration is 29.24sec.

(ii) For zero-polarized condensates ($M = 0$), all the \mathbf{P}_I^0 have nearly the same background $\mathbf{B}_I^0 \approx 1/2$ disregarding I as shown by eq.(9) of [24] and by Fig.1a of the present paper.

(iii) If N is even, I must be even so that M can be zero. When $I = 0$, starting from the initial state that half atoms are spin-up while the other half are spin-down, \mathbf{P}_0^0 goes up directly to $\mathbf{B}_I^0 \approx 1/2$ without oscillation as shown by the black curve in 1a. From this curve we know that half of the $\mu \neq 0$ atoms become $\mu = 0$ within $0.01t_p$, it certainly implies the occurrence of strong spin-flips $\uparrow + \downarrow \longleftrightarrow 0 + 0$. Then it remains extremely steady in a very long time until t is close to $t_p/4$, where a round of strong oscillation occurs suddenly.

(iv) When $I = 2$, a round of oscillation emerges in the early stage (red curve). When $I = 2k$, k rounds of oscillation emerge (blue curve). In general, \mathbf{P}_I^0 oscillates in the early stage but suddenly becomes very steady (this was first found in [5]), then the oscillation suddenly recovers. Therefore there are zones of oscillation (ZOO) and zones of steady evolution (ZOS). They appear alternately, namely, ZOO-ZOS-ZOO as shown by the dark cyan curve, and again. In general, when $I < N/2$, a larger I leads to a broader ZOO and accordingly a narrower ZOS. Incidentally, if N is odd, I is odd. When $I = 2k + 1 < N/2$, the ZOO contains $k + 1/2$ rounds of oscillation.

(v) When $I = N/4$, $N/8$ rounds of oscillation are contained in the ZOO (dark cyan curve). It turns out that, in this occasion, the duration of the ZOO is just $t_p/12$, that of the ZOS is also $t_p/12$. Therefore the two ZOO and the one ZOS of the dark cyan curve all have the same duration $t_p/12$. Meanwhile, the average frequency of oscillation in the ZOO is equal to $3N/(2t_p) \approx 3.14(N\omega^3)^{2/5} \times 10^{-5}$, which $\approx 5.13/\text{sec}$ for our parameters. It can be imagined that, when N is very large, the frequency would be very high and the ZOO would appear as a band with a width.

(vi) When I is close to $N/2$, the duration of the ZOO becomes very long (close to $t_p/8$), and the amplitude of oscillation becomes very small except in a small domain

close to $t_p/8$ as shown by the green curve in 1a. Accordingly, the ZOS in between becomes very narrow and unsteady.

(vii) When $I = N/2$, the two previous ZOOs transform to ZOSs, while the previous ZOS becomes to a very narrow ZOO containing only one round of oscillation situated at $t_p/8$ as shown by the top curve in 1a.

Fig.1a is for the cases with $I \leq N/2$. The cases with $I \geq N/2$ are given in Fig.3a. It was found that \mathbf{P}_I^0 and \mathbf{P}_{N-I}^0 are one-to-one roughly similar, however they are greatly different when t is close to 0 or $t_p/4$. The early stage of evolution is referred to Fig.2a and 4a.

IV. EVOLUTION UNDER A MAGNETIC FIELD

When $B \neq 0$, via the procedure of diagonalization, we obtain the following numerical results.

(I) Weak field

The evolutions of \mathbf{P}_I^0 are given in the (b) and (c) panels of Fig.1 to 4, we found

(i) The effect of B is very sensitive if I is close to 0 or N . Otherwise \mathbf{P}_I^0 is less affected. Note that \mathbf{B}_I^0 of the lowest three curves in 1a (3a) are much higher (lower) than their I/N . The effect of B is to pull them down (push them up) so that \mathbf{B}_I^0 are closer to I/N .

(ii) \mathbf{P}_0^0 and \mathbf{P}_N^0 are extremely sensitive to B . This is shown by the black curves in 1b and 3b. The black curve representing \mathbf{P}_0^0 in Fig.1a is greatly pulled down by the field when $B = 10$ (1b), and it becomes nearly a horizontal line close to zero when $B = 30$ (1c). Thus, for \mathbf{P}_0^0 , the spin-flips which occurs strongly in the early stage is severely suppressed by the field.

For \mathbf{P}_N^0 with all the atoms in $\mu = 0$ initially, we know from Fig.3 that 64% (74%) of the atoms would be changed to $\mu \neq 0$ at $0.008t_p$ ($0.005t_p$) if $B = 0$ ($B = 30$). Thus, instead of being suppressed, the strong spin-flips in the early stage are accelerated by B . Consequently, a strong oscillation is thereby induced. When $B=10$, the oscillation is stronger in the duration ($t_p/8, t_p/4$), refer to the black and red curves of 3b. When $B=30$, the oscillation is extremely strong through out nearly all the time (black curve of 3c), this is a noticeable point and we will return to it.

(iii) When I is neither close to 0 nor N , the effect of B would be to shorten the ZOO and enlarge the ZOS if $I < N/2$ (e.g., the duration of the ZOO of the dark cyan curve of 1c is $t_p/13.2$ instead of $t_p/12$ in 1a), or reversely if $I > N/2$ (e.g., refer to the pink curves in Fig.3). Another effect of B is to spoil the stability of the ZOS, and cause “irregular” oscillation in both ZOO and ZOS.

(iv) It seems that a new mode of oscillation might be caused by B . This is hinted, say, by the wavy pink curves in 3b, 3c, and the blue curve in 1c. This point is further studied below.

(v) The strict periodicity and the inherent symmetry exist no more if $B \neq 0$.

(II) Strong field

The early stage of evolution of \mathbf{P}_I^0 under a strong field is plotted in Fig.5 within the duration $(0, t_p/40)$. Comparing 5a with Fig.2c, we found that the curves with I close to zero are further suppressed so that $\mathbf{P}_I^0 \approx I/N$ together with a negligible oscillation. Comparing 5a with Fig.4c, the curves with I close to N keep their oscillation. The curves with I neither very small nor very large are relatively less affected, and they become more or less similar to each other as shown by the blue, pink, and navy curves of 5a. For these curves the division into ZOO and ZOS holds no more. \mathbf{B}_I^0 of all the curves in 5a are close to I/N .

When the field is even stronger as in 5b, a new mode of oscillation emerges, where the new patterns contain a series of pulses, each has an olivary shape and includes certain rounds of oscillation inside. The number of round would decrease if I increases. The amplitudes of the oscillation is larger if $I \approx N/2$, and it will be too small to be seen if $I \approx 0$ or N . If B increases further, the olives would become longer and thinner, and include more rounds. In the strong B limit the oscillation is completely suppressed and all \mathbf{P}_I^0 become just horizontal lines.

When B is sufficiently large, the first term of the Hamiltonian $g\hat{S}^2$ can be considered as a perturbation. The set of the first order perturbative solutions of the Hamiltonian is

$$|\Phi_I\rangle = |I\rangle + \frac{g}{2q}A_-(M, I)|I+2\rangle - \frac{g}{2q}A_+(M, I)|I-2\rangle \quad (7)$$

With this set, $\Psi(t)$ (eq.(3)) can be calculated analytically. Accordingly, we have

$$\mathbf{P}_I^M(t) \approx I/N + \frac{g^2}{Nq^2}[A_-^2(M, I)(1 - \cos \alpha t) - A_+^2(M, I)(1 - \cos \alpha' t)] \quad (8)$$

where $\alpha = [2q - g(4N - 8I - 6)]\omega$, $\alpha' = \alpha - 16g\omega$. Eq.(7) and (8) holds if $|\frac{g}{2q}A_{\pm}(M, I)| \ll 1$.

When $M = 0$ and $I = 0$, $A_+(0, 0) = 0$ and $A_-(0, 0) = \sqrt{2}N$, thus

$$\mathbf{P}_0^0(t) \approx 2\frac{Ng^2}{q^2}(1 - \cos \alpha t) \quad (9)$$

When $M = 0$ and $I = N$, $A_-(0, N) = 0$ and $A_+(0, N) = 2N$, thus

$$\mathbf{P}_N^0(t) \approx 1 - 4\frac{Ng^2}{q^2}(1 - \cos \alpha' t) \quad (10)$$

Both (9) and (10) imply a small oscillation around a horizontal line as shown in 5b, where the amplitude of the black curve is too small to be seen.

When I is neither close to 0 nor $N - M$, the sum of $A_-^2(M, I)$ and $A_+^2(M, I)$ is much larger than their difference. Thus we have

$$\begin{aligned} \mathbf{P}_I^0(t) \approx I/N + \frac{g^2}{Nq^2}[A_-^2(0, I) - A_+^2(0, I) - (A_-^2(0, I) \\ + A_+^2(0, I))\sin(\frac{\alpha + \alpha'}{2}t)\sin(\frac{\alpha - \alpha'}{2}t)] \end{aligned} \quad (11)$$

Since $\alpha + \alpha'$ is much larger than $\alpha - \alpha'$, (11) describes a high frequency oscillation included in a low frequency oscillation as shown in Fig.5b. This explains the origin of the olivary shapes. The factor $\sin(\frac{\alpha - \alpha'}{2}t)$ has a period $\pi/(4|g|\omega)$. Therefore the length of the olive would tend to $\pi/(8|g|\omega) = t_p/8$. Furthermore, $\mathbf{P}_I^0(t)$ would oscillate around $I/N + \frac{g^2}{Nq^2}[A_-^2(0, I) - A_+^2(0, I)]$, where the second term is small, as shown in (11) and in Fig.5b.

V. THE EFFECT OF THE CONFINEMENT POTENTIAL AND THE PARTICLE NUMBER

The above numerical results are obtained with $\omega = 3000$ and $N = 400$. From the formula of our previous paper [24] we know that the change of ω is simply equivalent to a change of the scale of time, i.e., all the curves of $\mathbf{P}_I^M(t)$ would remain unchanged except that the implication of t_p is changed with ω via the relation $t_p \propto \omega^{-6/5}$. On the other hand, since $g \propto \omega^{1/5}$, the decrease of ω will slightly increase the ratio q/g , therefore will amplify slightly the effect of B .

When N changes, the period is also changed due to $t_p \propto N^{3/5}$. When the time is re-scaled, the effect of N on the evolution is mild if $B = 0$. This is shown in Fig.6a where $N = 4000$. Fig.6a is very similar to Fig.1a although N is ten times larger (note that the time scales of these two figures are different). In particular, the ZOO and ZOS exist, and the rounds of oscillation contained in the ZOO is also $I/2$. Furthermore, the ZOO of the dark cyan curve in 6a has also the duration $(0, t_p/12)$, however 500 rounds of oscillations are included inside. Since $g \propto N^{-3/5}$, the increase of N will increase the ratio q/g , therefore will also amplify the effect of B . Comparing 6b with 1c, the black curve is more severely suppressed, the red and blue are more severely pulled down.

VI. SUMMARY

The effect of the magnetic field on the spin-evolution of zero-polarized ^{87}Rb condensates has been studied. The

following points are summarized:

(i) *Periodicity and inherent symmetry.* This important feature of \mathbf{P}_I^M , is spoiled by the field.

(ii) *Background.* The field pushes all the \mathbf{B}_I^0 from $\approx 1/2$ towards I/N .

(iii) *Zone of steady evolution.* The ZOS is no more highly steady. When B is stronger, the division into ZOO and ZOS does not hold.

(iv) *Sensitivity.* When I is close to 0 or N , \mathbf{P}_I^0 is highly sensitive to B . Otherwise, it is less sensitive. This is further shown in Fig.7. From this figure one can see that B causes oscillation in general. The frequency of oscillation will increase if B increases. The amplitude will firstly increase with B . However, when B exceeds certain values, the amplitude decreases with B . When B is sufficiently large, the amplitude will become so small that \mathbf{P}_I^0 looks just like the horizontal line I/N . This situation would occur if $B > 60, 320$, and 2000 mG, respectively, when $I = 0, N$, and $N/2$. It implies that the suppression of the amplitude is much easier to be fulfilled if $I \approx 0$ or N . On the other hand, in 7c with $I = N/2$, $\mathbf{P}_{N/2}^0$ is only weakly disturbed even if B is as large as 50mG (red curve), and the amplitude is still not very small even if B is as large as 450mG (orange curve).

(v) *Strong oscillation.* When I is close to N , a mediate field (roughly, $30 \leq B \leq 180$ mG) causes a particularly strong oscillation of \mathbf{P}_I^0 through out nearly all the time as clearly shown in Fig.7b. This is a noticeable point.

(vi) *New modes of oscillation.* Fig.7c shows clearly how the increase of B leads to the appearance of the new mode with olivary shape of pulses. That is similar to the quantum beats.

Acknowledgments

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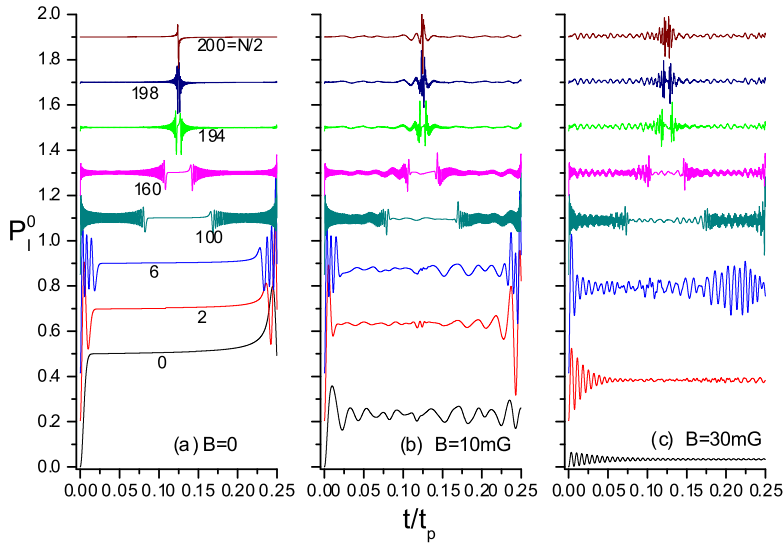


FIG. 1: (colored online) Evolution of $P_I^0(t)$ with various I and B . All figures in this paper are for ^{87}Rb atoms with $\omega = 3000$ and $N = 400$ (the only exception of N is Fig.6). Accordingly, the period $t_p = 117\text{sec}$, and t is given from 0 to $t_p/4$ (the case with $t > t_p/4$ can be understood from the inherent symmetry). I is from 0 to $N/2$ marked by the curves. Each curve has been shifted up by 0.2 more than its lower neighbor.

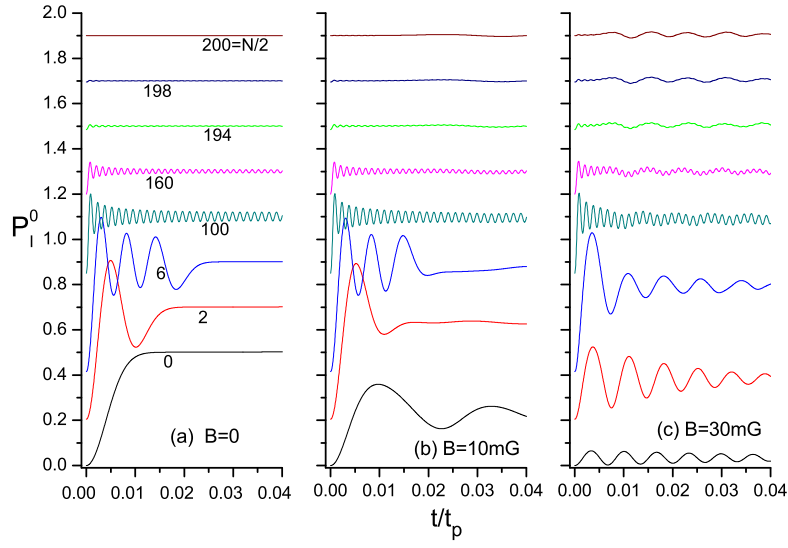


FIG. 2: (colored online) The same as Fig.1 but given only in the early stage.

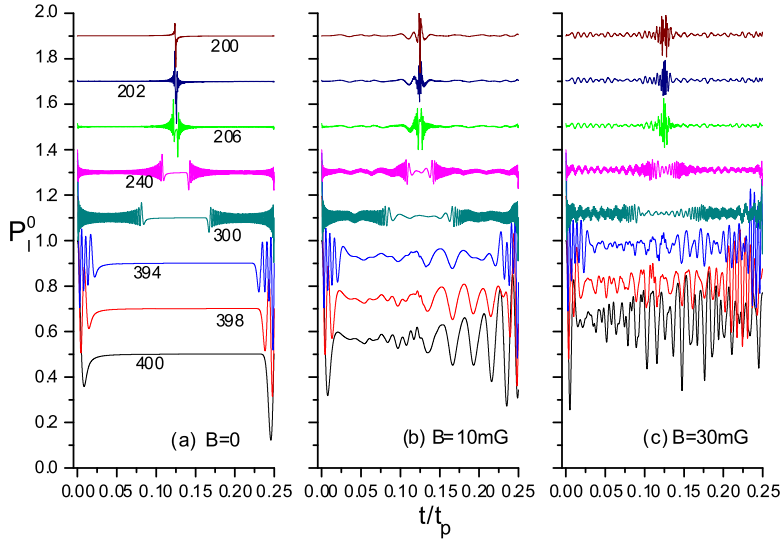


FIG. 3: (colored online) The same as Fig.1 but with I from $N/2$ to N .

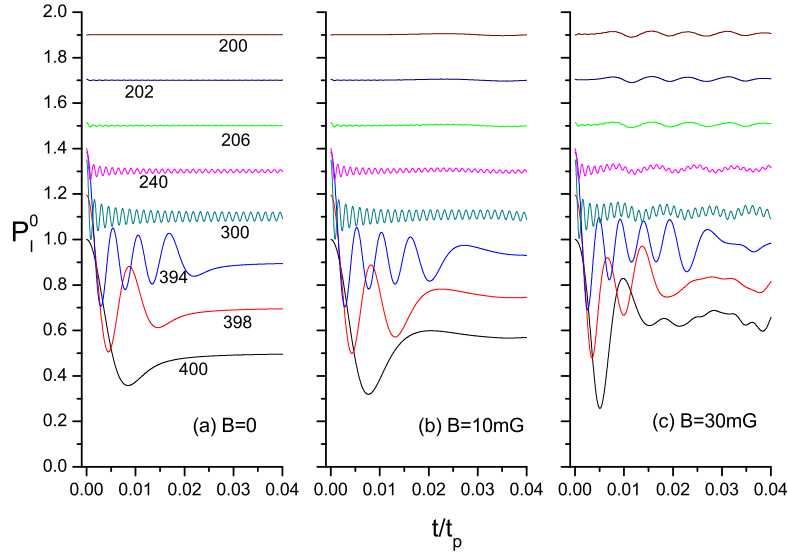


FIG. 4: (colored online) The same as Fig.3 but given only in the early stage.

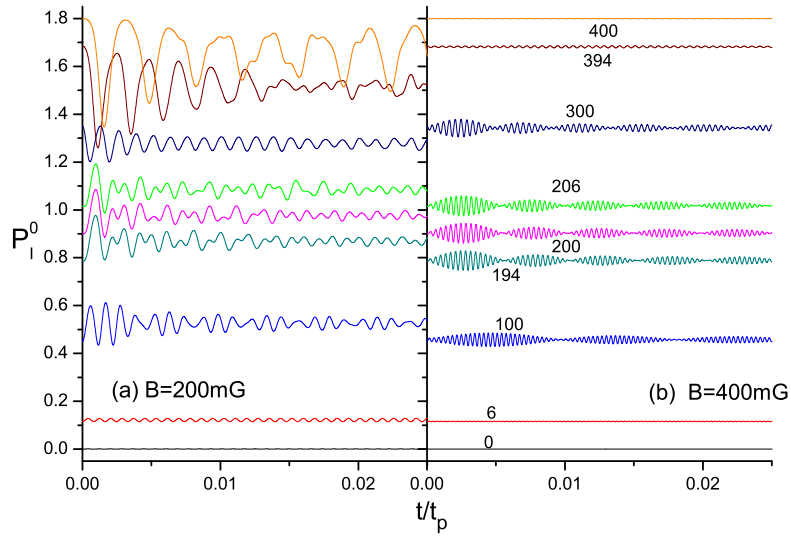


FIG. 5: (colored online) Early stage of evolution of $\mathbf{P}_I^0(t)$ under a strong B . I is from 0 to N marked by the curves. Each curve has been shifted up by 0.1 more than its lower neighbor. t is from 0 to $t_p/40$.

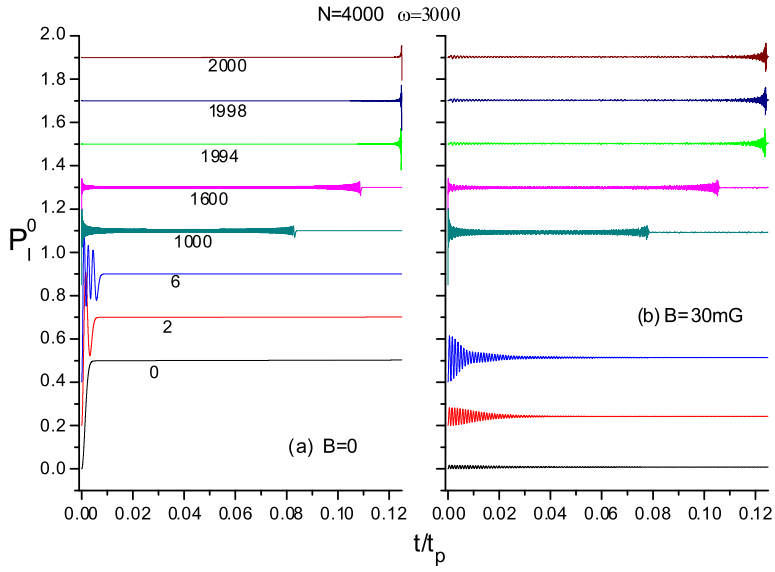


FIG. 6: (colored online) Similar to Fig.1 but with $N = 4000$. t is from 0 to $t_p/8$. A new set of I each is marked by the associated curve.

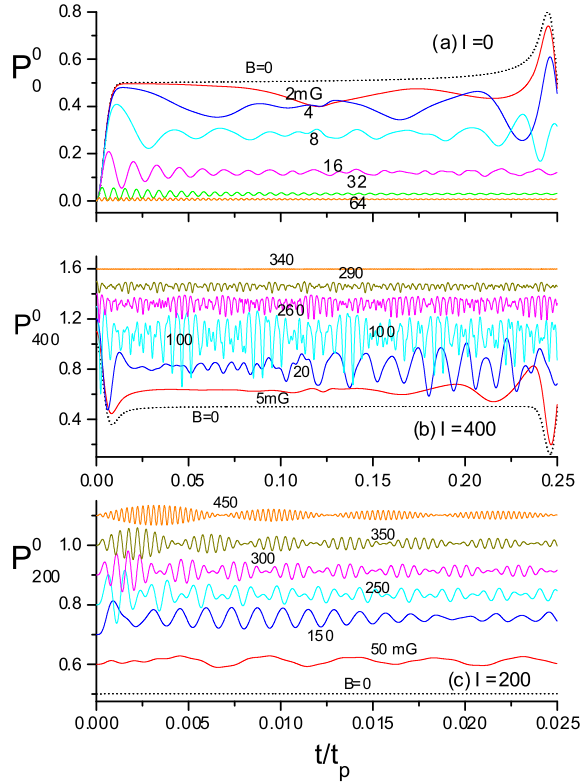


FIG. 7: Evolution of $\mathbf{P}_I^0(t)$ with various I and B . B is marked by the curves. The dotted lines are corresponding to $B = 0$. Each curve of (b) and (c) has been shifted up by 0.1 more than its lower neighbor. t is from 0 to $t_p/4$ (a and b) or to $t_p/40$ (c).